NASA
Technical Memorandum 102022
AIAA-89-2919

AVSCOM
Technical Report 89-C-007

Transmission Overhaul and Replacement Predictions Using Weibull and Renewal Theory

M. Savage
University of Akron
Akron, Ohio

and

D.G. Lewicki
Propulsion Directorate
U.S. Army Aviation Research and Technology Activity—AVSCOM
Lewis Research Center
Cleveland, Ohio

Prepared for the
25th Joint Propulsion Conference
cosponsored by the AIAA, ASME, SAE, and ASEE
Monterey, California, July 10–12, 1989

NASA

US ARMY
AVIATION
SYSTEMS COMMAND
AVIATION R&T ACTIVITY

TRANSMISSION OVERHAUL AND REPLACEMENT PREDICTIONS USING WEIBULL AND RENEWAL THEORY

M. Savage
The University of Akron
Akron, Ohio 44325

and

D. G. Lewicki
Propulsion Directorate
U.S. Army Aviation Research and Technology Activity - AVSCOM
Lewis Research Center
Cleveland, Ohio 44135

E-4576

Abstract

A method to estimate the frequency of transmission overhauls is presented. This method is based on the two-parameter Weibull statistical distribution for component life. A second method is presented to estimate the number of replacement components needed to support the transmission overhaul pattern. The second method is based on renewal theory. Confidence statistics are applied with both methods to improve the statistical estimate of sample behavior. A transmission example is also presented to illustrate the use of the methods. Transmission overhaul frequency and component replacement calculations are included in the example.

Nomenclature

b	Weibull slope
e	base of the natural logarithm
F	probability distribution function, probability of failure
F_k	probability of at least k failures
f	probability density function
Ln	natural logarithm
ℓ	life, hr
M	renewal function
M_e	approximate renewal function
N_r	number of replacements
Q	sample size
R	probability of survival, $(1 - F)$
x	integration time variable, hr
z_{10}	number of standard deviations from the mean which cuts off a 10-percent population tail
Γ	the gamma function
θ	characteristic life, hr
μ_3	third moment of a probability density function
σ	standard deviation

Subscripts:

av	average or mean
f	Weibull function
i	index
k	index
me	approximate renewal function
n	number of components in system
r	replacement function
s	system
10	90-percent reliability
90	90-percent confidence

Introduction

The in-flight service reliability of aircraft transmissions is much greater than the design reliability of their components. Transmission overhauls provide the difference. By monitoring the onset of potential transmission fatigue failures, just-in-time overhauls maintain the transmission economically.[1,2] One cause for propulsion system overhauls is the finite fatigue life of drive system components. The two-parameter Weibull distribution describes the statistics of drive system bearing and gear life.[3-5]

Component reliabilities and lives affect transmission maintenance costs which are significant. Estimates of these costs are important in the design stage of a transmission.[6] The two-parameter Weibull distribution provides information on component reliability and life. It does not predict overhaul frequency directly.

Two steps are required to convert component life statistics into overhaul frequency values. The first is the transmission system life model. This model is a two-parameter Weibull distribution for the transmission system life.[7,8] The second step is renewal theory.

Renewal theory is a statistical model which describes the maintenance cycle. The theory considers the ongoing sequence of: use, failure onset, repair, and return to use. For this sequence, renewal theory predicts the frequency of component replacement and the number of replacements needed to support the service maintenance schedule.[3-12]

Confidence theory complements these statistical methods with estimates of the likelihood of the predictions. Higher confidence levels require more spare parts to cover a greater range of possible situations.[10,11]

The purpose of the research presented is to provide a methodology for calculating transmission life and the number of component replacements. The paper presents the theories and applies them to a simple transmission to illustrate their use. Estimates of drive system and component lives and replacement needs are essential in design. These estimates provide a comparison of the relative worth of different designs from a safety and maintenance cost perspective. They also help assess the cost of operating a proposed drive system.

Component Life and Reliability

The two-parameter Weibull distribution is commonly used to describe fatigue life data. It can describe a wide variety of life patterns. The reliability of a component is the complement of its probability of failure.

In statistics, reliability is a double negative. Reliability or the act of surviving is the state of not having failed. Statistics count direct events such as the act of failing. A part can fail only once. It survives for its entire life. Thus the probability of failure is a direct statistic. The probability of failure for the two-parameter Weibull distribution is:

$$F = 1.0 - e^{-(\ell/\theta)^b} \tag{1}$$

where F is the probability of failure expressed as a decimal, e is the base of the natural logarithm, ℓ is the component life in million load cycles or hours, θ is the characteristic life in million load cycles or hours, and b is the Weibull slope. The two Weibull parameters are θ and b.

The derivative of Eq. (1) with respect to life is the probability density function, f:

$$f = \frac{b}{\theta}\left(\frac{\ell}{\theta}\right)^{b-1} e^{-(\ell/\theta)^b} \tag{2}$$

The probability density function is a histogram of life failures for a unit population. It presents the scatter in the component lives.

The Weibull reliability function is often expressed as:

$$\mathrm{Ln}\left(\frac{1}{R}\right) = \left(\frac{\ell}{\theta}\right)^b \tag{3}$$

where R is the probability of survival $(1 - F)$. For a 90-percent probability of survival, $R = 0.9$ and $\ell = \ell_{10}$. Solving for the characteristic life gives:

$$\theta = \mathrm{Ln}\left(\frac{1}{0.9}\right)^{-1/b} \ell_{10} \tag{4}$$

Figure 1 is a plot of the ratio of the characteristic life to the 90-percent reliability life as a function of the Weibull slope. Substituting θ of Eq. (4) in Eq. (3) gives:

$$\mathrm{Ln}\left(\frac{1}{R}\right) = \mathrm{Ln}\left(\frac{1}{0.9}\right)\left(\frac{\ell}{\ell_{10}}\right)^b \tag{5}$$

Equation (5) is the form used by manufacturers to present the two-parameter Weibull distribution characteristics of bearings.[13]

In both Eqs. (3) and (5), the logarithm of the reliability reciprocal is proportional to the life raised to the Weibull slope. Taking the logarithm of either equation generates a straight line plot as shown in Fig. 2. The plot is a probability graph for the two-parameter Weibull distribution.

This graph aids in determining the distribution parameter values for fatigue test data.[14] The plotted test data are the results of a series of identical life tests for a sample set of identical components. The first failure determines the highest reliability data point. The next failure determines the next lowest reliability data point, and so on.

The average life is the Mean Time to Failure (MTTF). It is the sum of all times to failure divided by the total number of the failures. The total number of failures for a continuous probability distribution is unity by definition. The sum of all times to failure is the integral of the product of time or life and the probability density function. The limits on the integral are from zero to infinity. The average life is:

$$\ell_{av} = \mathrm{MTTF} = \int_0^\infty \ell f(\ell)d\ell. \tag{6}$$

For a Weibull failure distribution, the solution to this integral involves the well known gamma function, Γ.[15] The solution is the gamma function multiplied by the characteristic life, θ.

$$\ell_{av} = \mathrm{MTTF} = \theta\Gamma\left(1 + \frac{1}{b}\right) \tag{7}$$

Figure 3 is a plot of the ratio of the average life to the 90-percent reliability life as a function of the Weibull slope. The average life equals the characteristic life when $b = 1$.

The standard deviation of a failure distribution is:

$$\sigma_f = \left[\int_0^\infty (\ell - \ell_{av})^2 f(\ell)d\ell\right]^{1/2} \tag{8}$$

In terms of the characteristic life, the Weibull slope, and the gamma function, the standard deviation of the two-parameter Weibull distribution is:

$$\sigma_f = \theta\left[\Gamma\left(1 + \frac{2}{b}\right) - \Gamma^2\left(1 + \frac{1}{b}\right)\right]^{1/2} \tag{9}$$

The standard deviation of a distribution is a measure of the scatter of the distribution. It is valuable in estimating a confidence limit for the average life. Figure 4 is a plot of the ratio of the standard deviation to the 90-percent reliability life as a function of the Weibull slope. At a slope of $b = 1$, the Weibull distribution is the exponential distribution and has a large scatter. As the slope increases, the scatter decreases rapidly.

System Life and Reliability

One model for the life of a drive system is the strict series probability model.[8] This model compares a system of load carrying gears and bearings to a chain of links. A chain fails when any single link fails. So too, a drive system requires repair when any component requires replacement or repair. In the strict series probability model, the reliability of a system, R_S, is the product of the reliabilities of all the components.

$$R_S = \prod_{i=1}^{n} R_i \tag{10}$$

The high speed of drive system components and the scattering of loose debris warrant the strict series probability model. If any component fails, debris may be present which could damage other components. Therefore, a drive system requires an overhaul to return it to a high state of reliability once any element fails.

Taking the logarithm of the reciprocal of Eq. (10) and using Eq. (5) for each component yields:

$$\operatorname{Ln}\left(\frac{1}{R_S}\right) = \operatorname{Ln}\left(\frac{1}{0.9}\right) \sum_{i=1}^{n} \left(\frac{\ell_S}{\ell_{i10}}\right)^{b_i} \tag{11}$$

In Eq. (11), ℓ_S is the life of the entire drive system for the system reliability, R_S. It is also the life of each component at the same drive system reliability, R_S. For consistency in Eq. (11), all the component lives must be defined in the same units. The unit chosen is hours.

Equation (11) is not a simple two-parameter Weibull relationship between system life and system reliability. The equation is a true two-parameter Weibull distribution only when all the Weibull exponents, b_i, are equal. In general, this is not the case. Thus, R_S as a function of ℓ_S when plotted such as in Fig. 2 may produce a curve rather than a straight line. A true two-parameter Weibull distribution can be approximated quite well, however, by fitting the curve using a least squares method.[8] The slope of the fitted straight line is the drive system Weibull slope, b_S. The life at which the drive system reliability equals 90 percent on the straight line is ℓ_{S10}. The drive system two-parameter Weibull relationship is then:

$$\operatorname{Ln}\left(\frac{1}{R_S}\right) = \operatorname{Ln}\left(\frac{1}{0.9}\right)\left(\frac{\ell_S}{\ell_{S10}}\right)^{b_S} \tag{12}$$

Renewal Theory

In addition to scheduling maintenance periods, a service procedure must also estimate the number of replacement components required. Renewal theory adds the renewal function to the statistical tools for estimating repair. It estimates the number of replacements as a function of a component failure distribution and its life.[9-11]

Renewal theory assumes the replacement of failed components when they fail. This models an unending sequence of use and repair. Aircraft drive system maintenance follows this pattern closely. The renewal function results from a sequence of statistically predicted failures.

Consider the maintenance sequence. In a given life period, any number of failures may occur. The probability of at least one failure within a given life from the start of operation is:

$$F_1(\ell) = F(\ell) = \int_0^{\ell} f(x)dx \tag{13}$$

The probability of at least two sequential failures in the period is the probability of two independent events. The first component must fail. Then a second component must begin its service life at this failure life and also fail. The probability of having at least two failures in this period is:

$$F_2(\ell) = \int_0^{\ell} F_1(\ell - x)f(x)dx \tag{14}$$

In Eq. (14), x is the time at which the first failure occurs. This can happen any time between zero and ℓ. At $x = 0$, the entire life is available for the first failure probability. The probabilities of the second failure and the combination event are both zero. As x increases from zero to ℓ, the probability of the first failure happening at time x decreases. The probability of the second failure increases. At $x = \ell$, the entire life is available for the second failure probability. The probability of the first failure is zero. The probability of the combined event is thus zero as well at $x = \ell$. The integral defines a function for the probability of at least two failures in the life period from zero to ℓ.

Equation (14) repeats indefinitely with increasing subscripts. The probability of having at least k failures in the period from zero to ℓ is:

$$F_k(\ell) = \int_0^{\ell} F_{k-1}(\ell - x)f(x)dx \tag{15}$$

In Eq. (15), $F_{k-1}(\ell - x)$ is the probability of having at least $k - 1$ failures in the period from zero to $\ell - x$. The probability of at least k failures increases as the number of failures decrease. Also, the more life available for a failure, the greater is the chance that it will occur.

The mean number of failures is the infinite sum of the probabilities of at least k failures in the life period, ℓ. This function, $M(\ell)$, is the renewal function. It is expressed as:

$$M(\ell) = \sum_{k=1}^{\infty} F_k(\ell) \tag{16}$$

Equations (15) and (16) yield the number of replacements needed to support a maintenance schedule. The solution involves a large series of convolution integrals. The equations apply to any failure distribution. However, the solution is not easy to obtain. Figure 5 shows the renewal function for a component with a two-parameter Weibull reliability. The component life has $\theta = 5000$ hr and $b = 1.5$. Tabulated solutions to the renewal function for the two-parameter Weibull distribution are available.[11,14]

An approximation for the renewal function from Ref. 10 is:

$$M_e(\ell) = \frac{\ell}{\ell_{av}} - \frac{\ell_{av}^2 - \sigma_f^2}{2\ell_{av}^2} \tag{17}$$

The approximation accuracy increases as ℓ increases. Equation (17) is an asymptote to the exact renewal function for low-scatter distributions. For high-scatter distributions it approximates the renewal function closely.

The standard deviation of the renewal function gives a measure of the scatter in replacement needs from one sample to the next. Figure 6 is a plot of the renewal function standard deviation versus life for a component which has a two-parameter Weibull reliability distribution. It has a characteristic life of 5000 hr and a Weibull slope of 1.5.

The approximation for the standard deviation of the renewal function is:[10]

$$\sigma_{me}(\ell) = \left[\frac{\sigma_f^2}{\ell_{av}^3}\ell + \left(\frac{\ell_{av}^2 + \sigma_f^2}{4\ell_{av}^4}\right)\left(3\ell_{av}^2 + 5\sigma_f^2\right) - \frac{2\mu_3}{3\ell_{av}^3}\right]^{1/2} \tag{18}$$

where μ_3 is the third moment of the life distribution. For the two-parameter Weibull distribution, the third moment is:

$$\mu_3 = \int_0^\infty \ell^3 f(\ell)d\ell = \theta^3\Gamma\left(1 + \frac{3}{b}\right) \tag{19}$$

Figure 7 is a plot of the ratio of the third moment to the 90-percent reliability life as a function of the Weibull slope.

Confidence Statistics

In predicting replacement rates and maintenance inventories, direct theory provides mean or average estimates. These estimates come from the statistics of a universal population (that is, an infinite number of samples). In any real situation, the number of drive systems under service is a limited sample. Confidence statistics estimate how differently a small sample may behave from its universal population. It uses the standard deviation of the universal failure distribution and the sample size to estimate the mean of the sample.[14]

For many samples of the same size, the mean of the samples has a normal distribution about the overall mean. The standard deviation of the means is:

$$\sigma_{av} = \frac{\sigma_f}{\sqrt{Q}} \tag{20}$$

where Q is the size of the sample.

The standard deviation of the number of replacements is:

$$\sigma_r = \sqrt{Q}\,\sigma_{me} \tag{21}$$

Also, the total number of replacements for the sample is:

$$N_r = QM(\ell) \tag{22}$$

In reliability predictions, the lower confidence bound has much significance in aircraft applications. Systems that are less reliable than the average are important to identify for safety and economical reasons. The confidence distribution estimates the mean life which will be lower than the mean life of a chosen percentage of all samples of a given size. This life is less than the mean life for the entire population. For a 90-percent confidence:

$$\ell_{av,90} = \ell_{av} - z_{10}\sigma_{av} \tag{23}$$

where z_{10} is the number of standard deviations below the mean which cuts off 10 percent of the population. For a normal distribution, $z_{10} = 1.282$.[4,14] 90 percent of the normal distribution lie above $\ell_{av,90}$ and 10 percent lie below $\ell_{av,90}$.

With a 90-percent confidence that the replacements will be less, the replacement estimate for a component from zero to life ℓ is:

$$N_{r,90} = N_r + z_{10}\sigma_r \tag{24}$$

Since the behavior of samples differs from the behavior of the "ideal" distribution, confidence estimates are helpful. With the confidence estimates, one can see during the design phase the effects of sample size on the life and replacement estimates.

Example

Mean Life

Consider the single mesh transmission shown in Fig. 8. Assume the 90-percent reliability lives for the bearings and gears are given as those shown in Table 1. Also assume a Weibull slope of 1.2 for the bearings and 2.5 for the gears. It is desired to determine the transmission mean life with 90-percent confidence for a fleet of $Q = 50$ aircraft.

From Eq. (7) or Fig. 3, the average lives were determined for each component. From Eq. (9) or Fig. 4, the standard deviations for each component were determined. The results are shown in Table 1.

The transmission system 90-percent reliability life was determined based on the component lives using Eqs. (11) and (12). Fitting Eq. (12) to Eq. (11) for this data yields a two-parameter Weibull slope of $b_s = 1.57$ for the transmission. The system 90-percent reliability life is $\ell_{s10} = 1060$ hr. From Eq. (4) or Fig. 1, the transmission characteristic life is $\theta = 4440$ hr. From Eq. (7) or Fig. 3, the transmission average life is $\ell_{av} = 3990$ hr. From Eq. (9) or Fig. 4, the standard deviation of the transmission life is $\sigma_f = 2600$ hr. Table 1 includes these results.

With this data, one can estimate the overhaul frequency for a fleet of similar aircraft. The mean life of a small sample of aircraft could be lower than that for an infinite population of aircraft. Confidence statistics estimate the sample properties from the universal properties.

Let us estimate the overhaul frequency for the fleet which use these transmissions with a 90-percent confidence that the frequency is lower. From Eq. (20), the standard deviation of the mean life's distribution is:

$$\sigma_{av} = \frac{2600}{\sqrt{50}} = 368 \text{ hr} \tag{25}$$

Using Eq. (23), the estimate of the transmission's mean life with a 90-percent confidence is:

$$\ell_{av,90} = 3990 - 1.282(368) = 3520 \text{ hr} \tag{26}$$

Number of Replacements

The renewal function serves to estimate the number of components one needs to support the maintenance pattern. Consider bearing number one in the single mesh transmission of Fig. 8 and Table 1. It is desired to determine the number of replacements required at any given time with 90-percent confidence for a sample of 50 aircraft.

From Table 1, the average life is $\ell_{av} = 16\,200$ hr and the standard deviation is $\sigma_f = 13\,560$ hr. The renewal function from Eq. (17) is:

$$M_e(\ell) = \frac{\ell}{16\,200} - \frac{(16\,200)^2 - (13\,560)^2}{2(16\,200)^2}$$

$$= \frac{\ell}{16\,200} - 0.150 \tag{27}$$

From Eq. (22), the total number of replacements from 0 hr to a life of ℓ is:

$$N_r = 50\left(\frac{\ell}{16\,200} - 0.150\right) = \frac{\ell}{324} - 7.5$$

From Eq. (19) or Fig. 7, the third moment of the bearing life distribution is $\mu_3 = 1.697\times10^{13}$ hr^3. From Eq. (18), the standard deviation of the renewal function for bearing one is:

$$\sigma_{me}(\ell) = \left\{\frac{(13\,560)^2}{(16\,200)^3}\,\ell + \left[\frac{(16\,200)^2 - (13\,560)^2}{4(16\,200)^4}\right]\right.$$

$$\left.\cdot\left[3(16\,200)^2 + 5(13\,560)^2\right] - \frac{2(1.697\times10^{13})}{3(16\,200)^3}\right\}^{1/2}$$

$$= \left(\frac{\ell}{23\,122} + 0.1038\right)^{1/2} \tag{29}$$

From Eq. (21), the standard deviation of the number of replacements is:

$$\sigma_r = \sqrt{50}\left(\frac{\ell}{23\,122} + 0.1038\right)^{1/2} = \left(\frac{\ell}{462} + 5.19\right)^{1/2} \tag{30}$$

With a 90-percent confidence that the replacements will be less, the replacement estimate for bearing one from zero to life ℓ using Eq. (24) is:

$$N_{r,90} = \left(\frac{\ell}{324} - 7.5\right) + 1.282\left(\frac{\ell}{462} + 5.19\right)^{1/2} \tag{31}$$

As an example, let us use this information to determine the number of replacements required for bearing one if the 50 aircraft are to operate up to $\ell = 10\,000$ hr. Note that if renewal theory is not used, the probability of failure from Eq. (1) is $F = 0.41$ for bearing one at $\ell = 10\,000$ hr. This would lead to $50(0.41) = 21$ bearing required. This method implies that bearings for 21 aircraft have failed and these aircraft are no longer in service. The renewal theory method, however, implies that the failed bearings are replaced and these aircraft are put back in service. For $\ell = 10\,000$ hr, Eq. (31) estimates the need for 30 bearings to support the overhaul needs of the 50 aircraft, compared to 21 using only the Weibull failure distribution.

Summary of Results

A method to estimate the frequency of transmission overhauls was presented. A second method was presented to estimate the number of replacement components needed to support the transmission overhaul pattern. Confidence statistics were applied with both methods to improve the statistical estimate of sample behavior.

The method to predict overhaul frequency is based on a two-parameter Weibull system life model. The relationship between the system life model and the component life models was presented. In addition, formulas for the mean and standard deviation of the two-parameter Weibull distribution were given.

Renewal theory was presented as a tool to estimate the number of component replacements in a transmission. Approximation formulas were given for the mean and standard deviations of the renewal function. These approximations are valid for the two-parameter Weibull distribution. Formulas for sample replacement rates were given in terms of the renewal function.

Single sided confidence theory was presented for the overhaul frequency and the component replacement rate estimates. A transmission example was also presented to illustrate the use of the methods. Transmission overhaul frequency and component replacement calculations were included in the example.

References

1. Koelsch, N.A., "Fault Detection/Location System for Intermediate and Tail Rotor Gearboxes," American Helicopter Society, Rotary Wing Propulsion System Specialists Meeting, Paper RWP-17, Williamsburg, VA, Nov. 1982.

2. DiPasquali, F., "Application of Quantitative Debris Monitoring to Gear Systems," AIAA Paper 88-2982, July 1988.

3. Barlow, R.E., and Proschan, F., *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, Inc., New York, 1975.

4. Kapur, K.C., and Lamberson, L.R., *Reliability in Engineering Design*, John Wiley & Sons, Inc., New York, 1977.

5. Goldberg, H., *Extending the Limits of Reliability Theory*, John Wiley & Sons, Inc., New York, 1981.

6. Ford, D., "Reducing the Total Cost of Ownership - A Challenge to Helicopter Propulsion System Specialists," Specialists Meeting on Rotary Wing Propulsion Systems, American Helicopter Society, Paper RWP - 6, Alexandria, VA, 1986.

7. Lewicki, D.G., Black, J.D., Savage, M., and Coy, J.J., "Fatigue Life Analysis of a Turboprop Reduction Gearbox," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 108, No. 2, June 1986, pp. 255-262.

8. Savage, M., Radil, K.C., Lewicki, D.G., and Coy, J.J., "Computerized Life and Reliability Modeling for Turboprop Transmissions," NASA TM-100918, 1988, *Journal of Propulsion and Power*, (at press).

9. McCall, J.C., "Renewal Theory - Predicting Product Failure and Replacement," *Machine Design*, Vol. 48, No. 7, Mar. 1976, pp. 149-154.

10. Cox, D.R., *Renewal Theory*, Chapman and Hall, New York, 1962.

11. White, J.S., "Weibull Renewal Analysis," *Proceedings of the Third Annual Aerospace Reliability and Maintainability Conference*, Society of Automotive Engineers, 1964, pp. 639-657.

12. Coy, J.J., Zaretsky, E.V., and Cowgill, G.R., "Life Analysis of Restored and Refurbished Bearings," NASA TN D-8486, 1977.

13. Harris, T.A., *Rolling Bearing Analysis*, 2nd Ed., John Wiley & Sons, Inc., New York, 1984.

14. Lipson, C., and Sheth, N.J., *Statistical Design and Analysis of Engineering Experiments*, McGraw-Hill Book Co., New York, 1973.

15. Abramowitz, M., and Stegun, I.A., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series #55, Washington, D.C., June 1964.

TABLE 1. - SINGLE MESH TRANSMISSION PROPERTIES

	ℓ_{10}, hr	ℓ_{av}, hr	σ_f, hr
Bearing 1	2640	16 200	13 560
Bearing 2	4820	29 570	24 750
Pinion	2480	5 410	2 320
Bearing 3	7230	44 360	37 130
Bearing 4	3960	24 300	20 330
Gear	3170	6 920	2 960
Transmission	1060	3 990	2 600

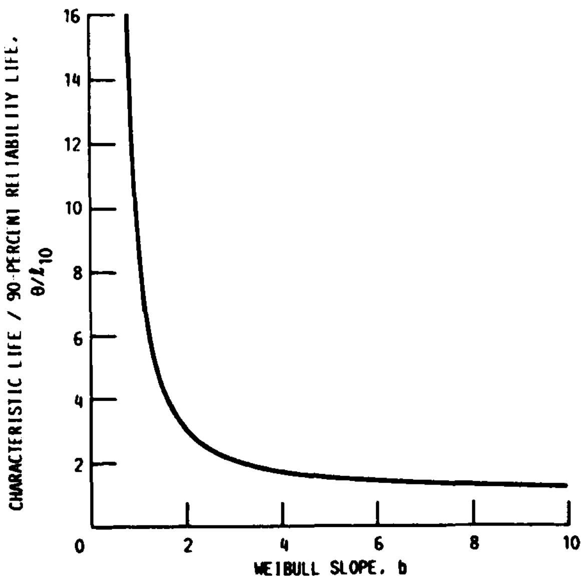

FIGURE 1. - CHARACTERISTIC LIFE TO 90-PERCENT RELIABILITY LIFE RATIO FOR A WEIBULL DISTRIBUTION AS A FUNCTION OF THE WEIBULL SLOPE.

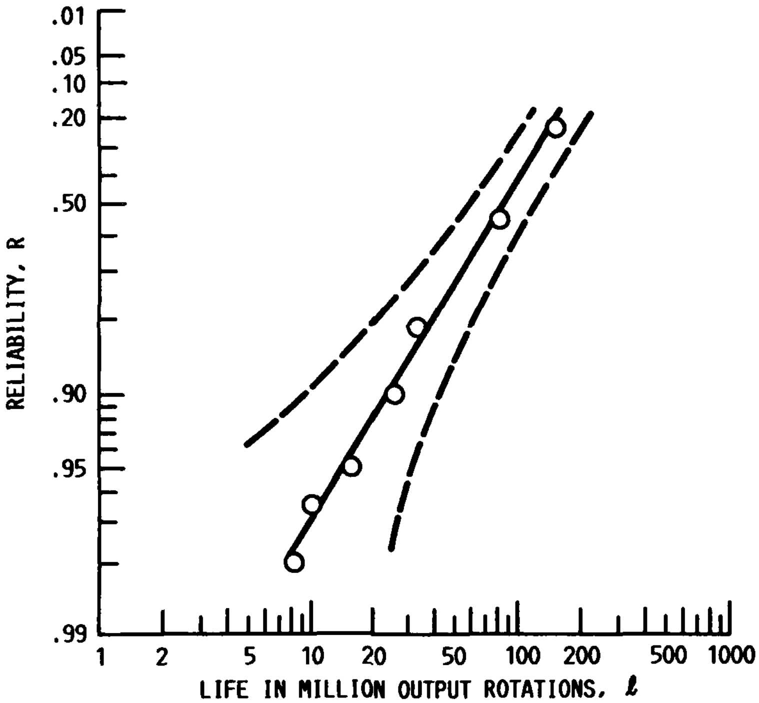

FIGURE 2. - TWO-PARAMETER WEIBULL PROBABILITY PLOT.

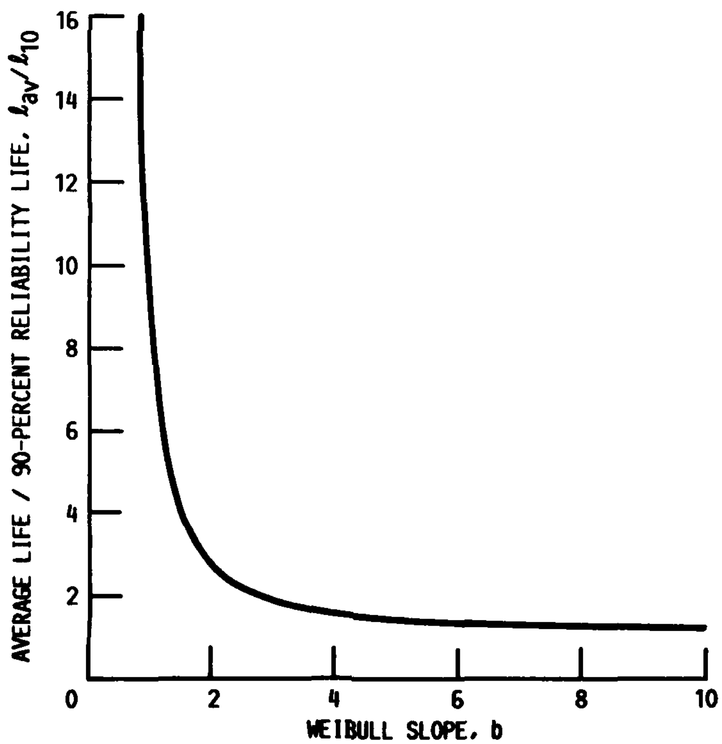

FIGURE 3. - AVERAGE LIFE TO 90-PERCENT RELIABILITY LIFE RATIO FOR A WEIBULL DISTRIBUTION AS A FUNCTION OF THE WEIBULL SLOPE.

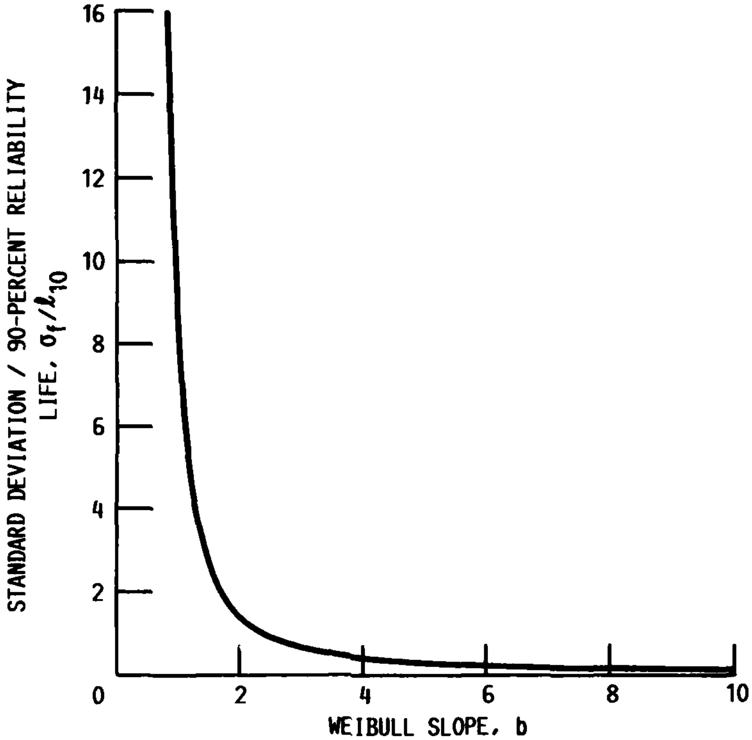

FIGURE 4. - STANDARD DEVIATION TO 90-PERCENT RELIABILITY LIFE RATIO FOR A WEIBULL DISTRIBUTION AS A FUNCTION OF THE WEIBULL SLOPE.

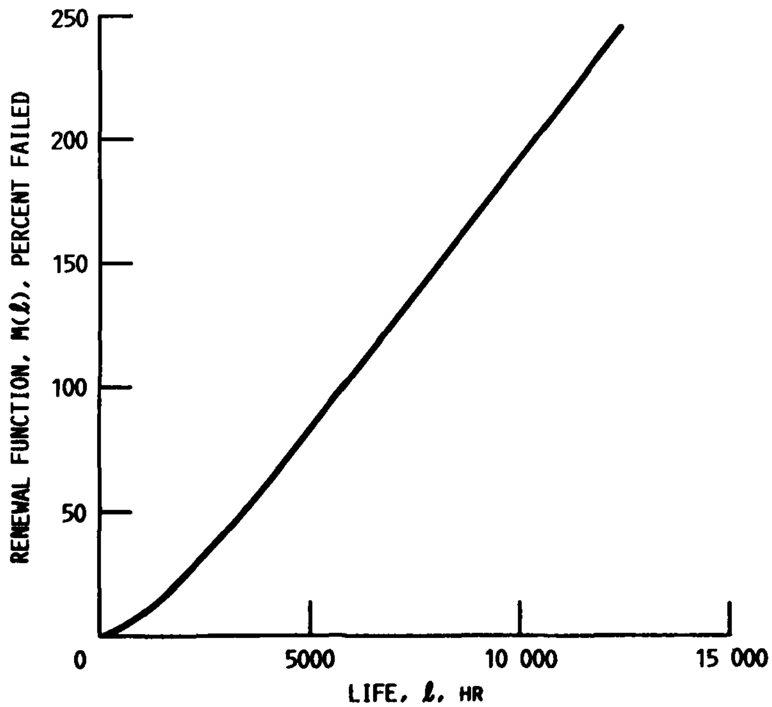

FIGURE 5. - RENEWAL FUNCTION FOR A TWO-PARAMETER WEIBULL DISTRIBUTION WITH θ = 5000 HOURS AND b = 1.5.

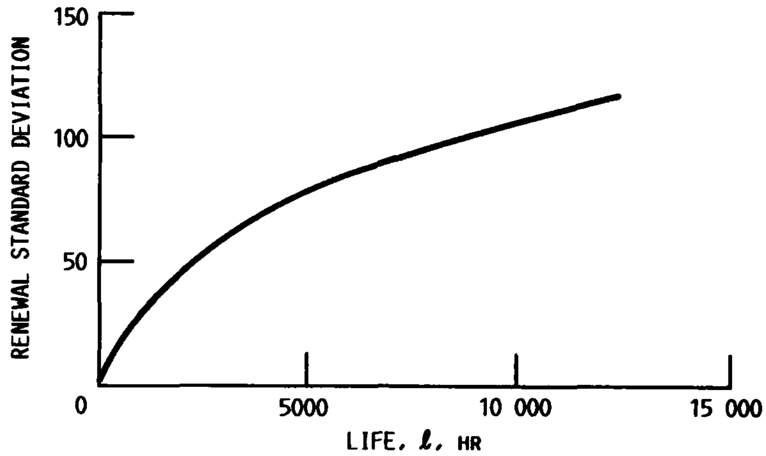

FIGURE 6. - RENEWAL FUNCTION STANDARD DEVIATION FOR A TWO-PARAMETER WEIBULL DISTRIBUTION WITH θ = 5000 HOURS AND b = 1.5.

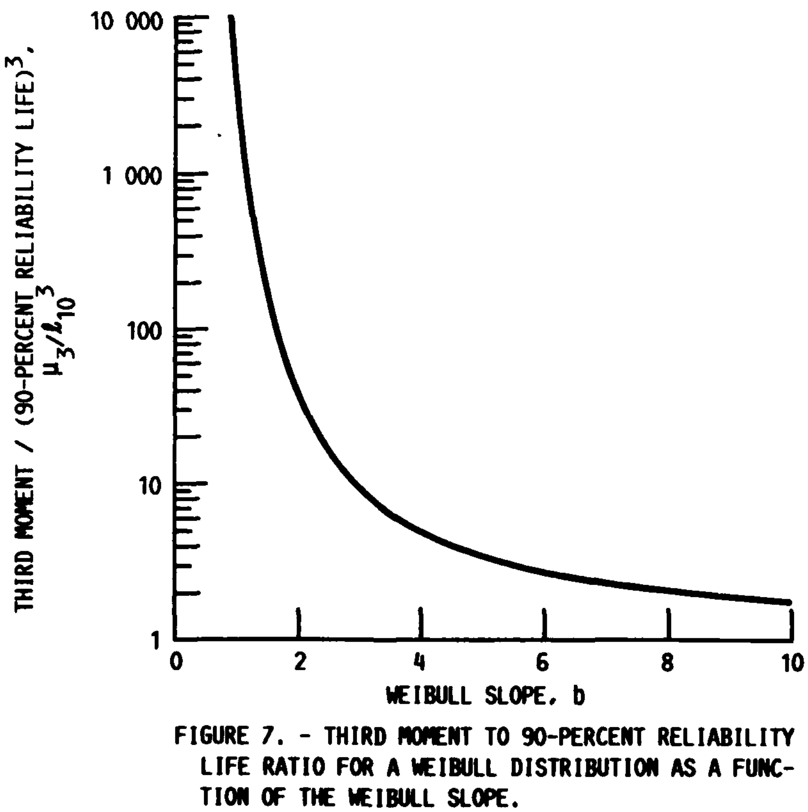

FIGURE 7. - THIRD MOMENT TO 90-PERCENT RELIABILITY LIFE RATIO FOR A WEIBULL DISTRIBUTION AS A FUNCTION OF THE WEIBULL SLOPE.

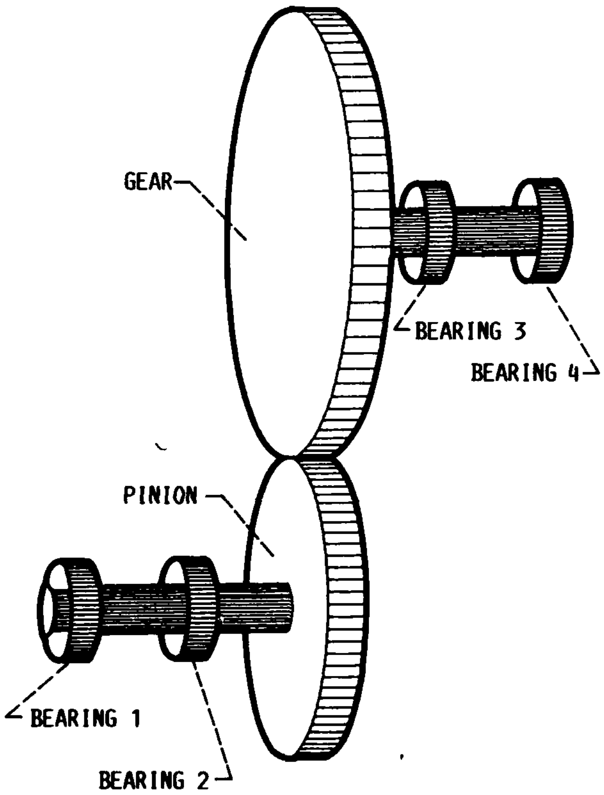

FIGURE 8. - SINGLE MESH TRANSMISSION EXAMPLE.

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No.
NASA TM-102022
AVSCOM TR 89-C-007; AIAA-89-2919

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
Transmission Overhaul and Replacement Predictions Using Weibull and Renewal Theory

5. Report Date

6. Performing Organization Code

7. Author(s)
M. Savage and D.G. Lewicki

8. Performing Organization Report No.
E-4756

9. Performing Organization Name and Address
NASA Lewis Research Center
Cleveland, Ohio 44135-3191
and
Propulsion Directorate
U.S. Army Aviation Research and Technology Activity—AVSCOM
Cleveland, Ohio 44135-3127

10. Work Unit No.
505-63-51
1L162209A47A

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546-0001
and
U.S. Army Aviation Systems Command
St. Louis, Mo. 63120-1798

13. Type of Report and Period Covered
Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes
Prepared for the 25th Joint Propulsion Conference cosponsored by the AIAA, ASME, SAE, and ASEE, Monterey, California, July 10–12, 1989. M. Savage, University of Akron, Akron, Ohio 44325; D.G. Lewicki, Propulsion Directorate, U.S. Army Aviation Research and Technology Activity—AVSCOM.

16. Abstract
A method to estimate the frequency of transmission overhauls is presented. This method is based on the two-parameter Weibull statistical distribution for component life. A second method is presented to estimate the number of replacement components needed to support the transmission overhaul pattern. The second method is based on renewal theory. Confidence statistics are applied with both methods to improve the statistical estimate of sample behavior. A transmission example is also presented to illustrate the use of the methods. Transmission overhaul frequency and component replacement calculations are included in the example. Keywords:

17. Key Words (Suggested by Author(s))
Weibull failure distribution; Renewal theory; Transmissions; Gears; Bearings; Overhauls,

18. Distribution Statement
Unclassified – Unlimited
Subject Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No of pages	22. Price*
Unclassified	Unclassified	12	A03

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161